

# Optical activity of a neutrino sea in the Standard Model

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## Abstract

We present the Standard Model calculation of the optical activity of a neutrino sea.

The idea that intergalactic space is a birefringent medium for light due to the presence of a neutrino sea has been contemplated for a long time. Some thirty years ago, Royer [1] computed in  $V - A$  theory an effect of order  $G_F$ . Later, Stodolsky noted that due to a theorem of Gell-Mann [2], there can be no such effect with massless neutrinos and a point-like coupling, and his observation was recorded by one of the present authors in a review [3]. In the early 1980's data on propagation of radio waves through intergalactic Space put a stringent upper bound on possible optical activity of the neutrino sea [4] and [5] and this led to renewed estimates for the size of such effect on the assumption of a neutrino magnetic moment, which occur for a massive neutrino [5]. More recently [6] an evaluation was made for a off-mass shell photons within the Standard Model (SM). Here we note that there is a frequency dependent effect even for real photons and we also comment on the effect for virtual photons (both within the SM). Thus we shall consider three possible effects: a magnetic moment effect, an effect for virtual photons, an effect for real photons.

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If the neutrino has a magnetic moment, this moment will interact with the magnetic field of an electromagnetic wave:

$$\mathcal{L}_{int} = (e\mu/2m)\bar{\nu}(p_2)\sigma_{\alpha\beta}q_{\beta}\nu(p_1)\epsilon(q) \quad , \quad (1)$$

In this equation  $\epsilon(q)$  is the polarization vector of a photon of momenta  $q$ , while  $\nu(p)$  is the neutrino bispinor, and  $\mu$  is the neutrino magnetic moment in Bohr magnetons  $e/2m$ .

With this interaction (1) the forward scattering amplitude of a photon of momentum  $k$  from a neutrino of momentum  $p$  is equal to

$$T = 4i(e\mu/m)^2\epsilon_{\mu\nu\alpha\beta}\epsilon_{\mu}(k)\epsilon_{\nu}^*(k)p_{\alpha}k_{\beta} \quad . \quad (2)$$

This amplitude differs for left-handed and right-handed photons:

$$T_{LL} = -T_{RR} = 4(e\mu/m_e)^2(pk) \quad , \quad (3)$$

and gives rise to a nonzero optical activity for a polarizes neutrino gas.

In the Standard Model the neutrino has no intrinsic magnetic moment, but a magnetic moment can emerge as a one loop effect if the neutrino is massive(See figure 1).

The order of magnitude of the magnetic moment is:

$$\mu = C(e^2/8\pi^2s^2)(m_em_{\nu}/m_W^2) \quad , \quad (4)$$

where  $s = \sin\theta_W$  and  $\theta_W$  is the weak mixing angle;  $s^2 \simeq 0.2312$ . Constant  $C$  can be obtained by direct calculation [7]

$$C = 3/4 \quad . \quad (5)$$

Thus a nonzero optical activity appears in this case as a two-loop effect

$$T_{LL} - T_{RR} = (9e^6/128\pi^4s^4)(pk/m_W^2)(m_{\nu}/m_W)^2 \quad . \quad (6)$$

It was understood long ago that  $\gamma\nu$  scattering takes place as a one-loop process even for massless neutrino. The first estimates were performed in the Fermi theory but erroneously [8]. According to Gell-Mann's theorem [2] the amplitude should vanish for point-like interactions. Somewhat later Levine [9] evaluated the scattering mediated by W boson exchange and found a

nonzero amplitude ( See figure 2). Due to gauge invariance the amplitude starts with the second power of the momentum of the photon  $k$ . Levine computed the first non-vanishing term, which in the SM looks like:

$$T = C(e^2/8\pi^2 s^2)(pk/m_W^2)^2 \epsilon \epsilon^* . \quad (7)$$

This amplitude is even under parity, in other words it is the same for right-handed photons and left-handed photons, so it cannot contribute to optical activity.

Very recently it has been found that by expanding to the next term of order  $k^3$  one finds non-vanishing P-odd terms. The result of this calculation is [6]:

$$T = (e^4/8\pi^2 s^2) \epsilon_{\mu\nu\alpha\beta} \epsilon_\mu(k) \epsilon_\nu^*(k) (p_\alpha k_\beta / m_W^2) (k^2 / 3m_e^2) . \quad (8)$$

This amplitude vanishes for real photons but is non-vanishing for virtual photons. The amplitude is singular with respect to the electron mass and comes entirely from the diagram 2a.

It is interesting to ask whether one can get a parity violating amplitude for real photons. We find a positive answer to this question. In the same order of expansion over small momenta  $k$  ( $k \ll m_W$ ) one finds the amplitude:

$$T = C(e^4/8\pi^2 s^2)(pk/m_W^2)^2 \epsilon_{\mu\nu\alpha\beta} \epsilon_\mu(k) \epsilon_\nu^*(k) (p_\alpha k_\beta / m_W^2) , \quad (9)$$

where

$$C = 4/3(\ln(m_W^2/m^2) - 11/3) . \quad (10)$$

This does not vanish for real photons and contributes to the optical activity of neutrino gas. The logarithmic term in equation (10) comes from diagram 2a, while the constant receives contributions from all diagrams of figure 2. The calculation is straightforward.

Before we discuss the order of magnitude of the different amplitudes we note that the authors of reference [6] have missed one diagram which contributes to off-shell photons. This is the diagram with  $Z$  exchange shown in Figure 3. For real photons,  $k^2 = 0$  the triangle diagrams with different fermions inside the loop cancel each other, so that the Standard Model is anomaly free. But for off-shell photons each diagram gives a contribution

proportional to  $k^2/m^2$ , where  $m$  is the mass of the fermion running inside the loop. The main contribution comes from the electron loop and is equal to  $(-1/2)$  of the contribution from diagram 2a (See ref. [10]). The sum of contributions from Figure 2a and Figure 3 gives

$$T = (e^4/8\pi^2 s^2) \epsilon_{\mu\nu\alpha\beta} \epsilon_\mu^*(k) \epsilon_\nu^*(k) (p_\alpha k_\beta / m_W^2) (k^2 / 6m_e^2) . \quad (11)$$

Summarizing these results, we obtain the order of magnitude estimates:

$$T_{LL} - T_{RR} = \Delta_1 \sim (\alpha^3/s^4) (pk/m_W^2) (m_\nu^2/m_W^2) \text{ for } \mu \neq 0, k^2 = 0; \quad (12)$$

$$\Delta_2 \sim (\alpha^2/s^2) (pk/m_W^2) (k^2/m_e^2) \text{ for } \mu = 0, k^2 \neq 0; \quad (13)$$

$$\Delta_3 \sim (\alpha^2/s^2) (pk/m_W^2)^3 \ln m_W^2/m_e^2 \text{ for } \mu = 0, k^2 = 0. \quad (14)$$

At low frequencies the scattering due to magnetic moment dominates even though it takes place at two-loop level:

$$(\Delta_1/\Delta_3) \sim (\alpha/s^2) (m_W/\omega)^2 / \ln m_W^2/m_e^2 \sim (1.7 \text{ GeV}/\omega)^2 . \quad (15)$$

Here we assume the neutrino at rest, i.e.  $pk = m_\nu \omega$ .

As for the relative value of  $\Delta_2$  and  $\Delta_3$

$$(\Delta_2/\Delta_3) \sim (m_W^2/pk)^2 (k^2/m_e^2) / \ln m_W^2/m_e^2 . \quad (16)$$

This is sensitive to the  $k^2$  value of the virtual photon. Following the authors of reference [6] we take for intergalactic space

$$k^2 = \omega_p^2 = en_e/m_e . \quad (17)$$

and assuming  $n_e \sim 0.03 \text{ cm}^{-3} \sim 2.3 \cdot 10^{-43} \text{ GeV}^3$ , we obtain:

$$(\Delta_2/\Delta_3) \sim (6 \text{ keV}/\omega)^2 (1 \text{ eV}/m_\nu)^2 . \quad (18)$$

For visible photons the on-shell activity is smaller than for off-shell photons, but in the  $X$ -ray region the reverse is the case.

In principle the contribution  $\Delta_3$  is similar to the rotation from the intergalactic magnetic fields, but the frequency dependence is very different. A numerical estimate shows that the contribution from the neutrino sea is many order of magnitude smaller than the magnetic ( Faraday ) effect, observed

with radio waves [11]. We estimate, for radio waves and for  $1eV$  neutrino sea with Fermi momenta  $k_F \sim 10^{-2}eV$  an effect of order  $10^{-82}$  *radians/meter* which is insignificant compared to the Faraday effect. At shorter wavelengths the rotatory power due to neutrino sea would increase. These small amplitudes are matched by very small cross sections for photon-neutrino scattering [12].

The scattering of laser photons with high energy intensive neutrino beam is discussed in the literature ( e.g. see [12] ). For this case the considered one-loop amplitude dominates over scattering due to magnetic moment:  $(\Delta_1/\Delta_3) \sim \alpha(m_W^2/pk)^2(m_\nu^2/m_W^2) \sim (m_\nu/1eV)^2 \cdot (60keV/\sqrt{s})^4$ .

## 1 Acknowledgments

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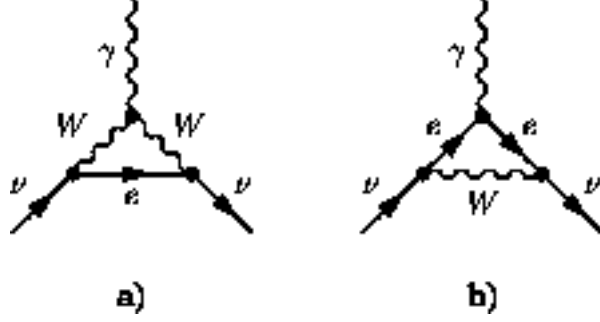


Figure 1: Neutrino magnetic moment in the Standard Model.

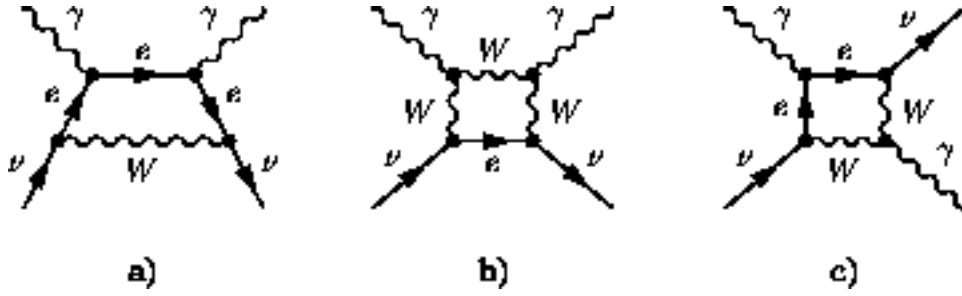


Figure 2: Diagrams contributing to  $P$ -odd  $\gamma\nu$  on-shell scattering.

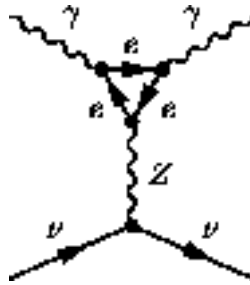


Figure 3: Diagram contributing to  $P$ -odd  $\gamma\nu$  off-shell scattering.